Criteria for Multiexponential Relaxation of Exchanging Spin 3/2 Nuclei.

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Introduction:

The problem of exchange of spin 3/2 nuclei between slowly rotating site and a site in the extreme narrowing limit has been studied by examining NMR lineshapes (1). This work was motivated by the expected effects of binding of sodium to macromolecules in biological systems (2-5). It was demonstrated (1) that these lineshapes are very sensitive to the presence of minute amounts of free sodium($\omega_0\tau_R$ <<1 where ω_0 is Larmour frequency, τ_R reorientation time). In the present work we consider the more common situation in biological systems where only small fraction of the sodium ions is bound to macromolecules. By solving the Liouville equation without perturbation approximation (1) we extend the study of lineshape in two ways a) slow motion effects b) exchange times are shorter than reorientation time (6). Using the theory of Ref.1 we set a quantitative criteria for the validity of second order perturbations theories(SOPT).

Results and discussion:

Perturbation theories predict that the transverse component of the magnetization will decay biexponentialy (7-9). Thus, it is customary to analyze relaxation in these terms. We would like to examine the conditions under which, (A) the numerical solutions of Liouville equation (1) may be fitted to sum of two Lorentzians, and (B) when the widths and dynamic shifts of these Lorentzians are described adequately by the analytical expressions given by the SOPT. This is done for two type of problems. In one, the spins are located in the same type of surrounding, and in the second they are exchanging between two different sites.
In order to get a quantitative measure of the fit of the numerical solutions, $M^n(\omega)$ to the analytical ones, $M^a(\omega)$, we define:

$$\text{FIT} = \frac{\max_{\omega}\left\{M_c^n(\omega)-M_c^a(\omega)\right\}}{\max_{\omega}\left\{M_c^n(\omega)\right\}} \qquad c=(y,z) \qquad [1]$$

To demonstrate the meaning of FIT we show in Fig.1 a spectrum with FIT=0.07

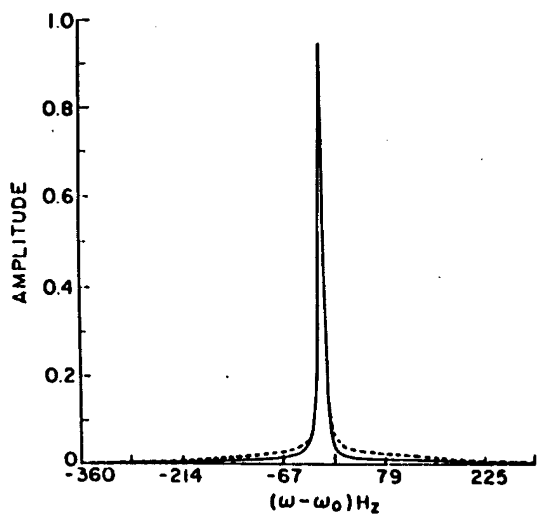

Fig.1 A comparison of a spectrum calculated by expressions given in (7) (full line) and the spectrum calculated by the numerical solution of Liouville equation (1) (dotted line). The amplitudes were adjusted to give a non-linear least mean squares fit. The value of FIT obtained for these spectra is 0.07. The parameters used in both type of calculations are: χ_I=0.6MHz τ_R^I=1X10^{-11}s χ_S=2MHz τ_R^S=4X10^{-6}s P_S=0.01 τ_S=10^{-5}s.

A. Fit to two arbitrary Lorentzians.

The numerical calculations of lineshapes were fitted to two Lorentzians (with widths $1/T_{2s}$ $1/T_{2f}$ and dynamic shifts Q_{2s}, Q_{2f}) using a non linear least mean squares procedure

(NLLMS) and FIT as criterion for the fit. For a single site case we obtain FIT≤0.032 for $\tau_R \leq 5X10^{-7}$s and $\chi \leq 25$MHz.
For exchanging sites the following range of values [2] for the exchange times τ_J (J=I,S), reorientation times τ_R^J, quadrupole interactions χ_J, and percentage P_S of bound sodium (to the macromolecule) were considered:

$$\tau_S \leq 10^{-1}s,\ \chi_S \leq 25MHz,\ \tau_R^S \leq 4X10^{-6}s,\ P_S \leq 28\% \qquad [2]$$

$$\tau_I \leq 10^{-1}s,\ \chi_I \leq 2MHz,\ \tau_R^I \leq 3X10^{-10}s,\ P_I > 72\%$$

and we find FIT≤0.012 as a result of slow motion. In order to study the effect of the exchange process on the parameters $1/T_{2f}$ and $1/T_{2s}$ obtained by the procedure described above, we calculated these parameters as a function of the exchange rate defined by $\lambda = 1/\tau_I + 1/\tau_S$, for various values of reorientation times. The plot of $1/T_{2f}$ (Fig.2) indicates one maximum for rotational times ranging from slow to fast motion. The maximum is obtained at $\lambda = \chi_S/P_I$.

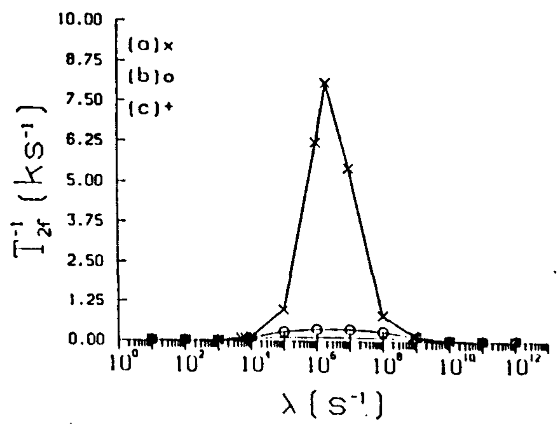

Fig.2 $1/T_{2f}$ as a function of λ for three values of τ_R^S. (a) $\tau_R^S = 3X10^{-7}$s (X) ; (b) $\tau_R^S = 4X10^{-9}$s (O) ; (c) $\tau_R^S = 7X10^{-10}$s (+) Other parameters are : $\chi_I = 0.6$MHz $\tau_R^I = 1X10^{-11}$s $\chi_S = 2$MHz $P_S = 1\%$.

On the other hand the plot of $1/T_{2s}$ shows a more complicated behavior. Under the condition $\omega_0 \tau_R^S > 10$ two maxima are obtained, one at $\lambda = 1.24\omega_0$ and another at $\lambda = Q_{2S}/P_I$. As the motion is getting faster the two maxima coalesce and for $\omega_0 \tau_R^S < 0.2$ one maximum is obtained at $\lambda = \chi_S/P_I$ (Fig.3c).

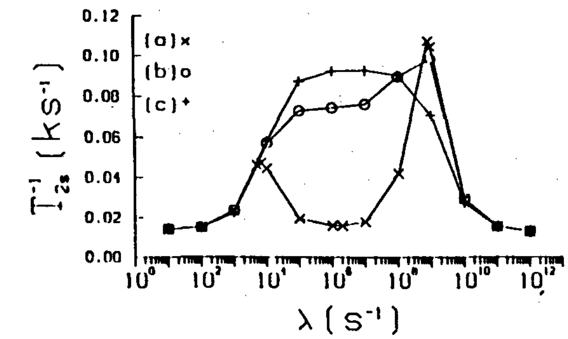

Fig.3 $1/T_{2s}$ as a function of λ. For explanation of the plots see caption for Fig.2

The maxima shown in Figs. 2 and 3 suggest the following explanation for the dependence of the linewidths on the exchange rate. Under the condition $\omega_0 \tau_R^S > 10$ the difference of the resonance frequencies of $1/2 \longleftrightarrow -1/2$ transitions of two sites is determined by the second order shift, Q_{2S}. Thus one expects maximum line broadening at about $\lambda = Q_{2S}$. Similarly, for the $\pm 1/2 \longleftrightarrow \pm 3/2$ transitions maximum line broadening is obtained at an exchange rate equals to the frequency difference between these transitions in the two sites which is equal to the quadrupole interaction χ_S.

B. An assessment of the validity of the analytical expressions derived from the second order perturbation theory.

In this section we would like to set the conditions under which the available analytical expressions (7-9) are applicable. we examine two cases, one case is a single site and the other one is of spins exchanging between two sites. For the single site problem analytical expressions for the lineshape (8-9) presents superposition of two Lorentzians. Following previous works (10-11) we define $\kappa = (\chi/\omega_0)^2 6\omega_0 \tau_R$ as a quantitative measure of the motion. We find three ranges of interest for κ. (a) fast motion, $\kappa \leq 1$, for which a good fit (FIT≤0.01) is obtained to the analytical expressions given in (9). (b) $1 < \kappa \leq 125$ where the lineshape fits a sum of two Lorentzians (FIT≤0.03) though their widths and peak positions differ significantly from the values predicted by SOPT(9) (c) $\kappa > 125$, where more than two Lorentzians are required to describe the spectrum. It seems of interest to find out how the microscopic parameters χ and τ_R which are conventionally extracted from two arbitrary Lorentzians are affected by the fact that $1 \leq \kappa \leq 125$ and it is still possible to get a good fit to two Lorentzians. Thus, we have extracted the microscopic parameters by two methods: One is based on the linewidths and solving the pair of equations for χ and τ_R (8). The other method is application of NLLMS to fit the numerical results to analytical expressions resulting from SOPT (9) leaving χ and τ_R as the variable parameters. The second of the methods takes into account the effect of dynamic shifts. For $\kappa \leq 25$ we obtained FIT<0.026. The two methods give similar results for $\kappa<25$ whereas for $\kappa \geq 25$ there are significant differences. In Fig.4 the ratio of τ_R used in the numerical calculation (τ_R(NC)) to its values as retrieved by the two methods is given as function of κ.

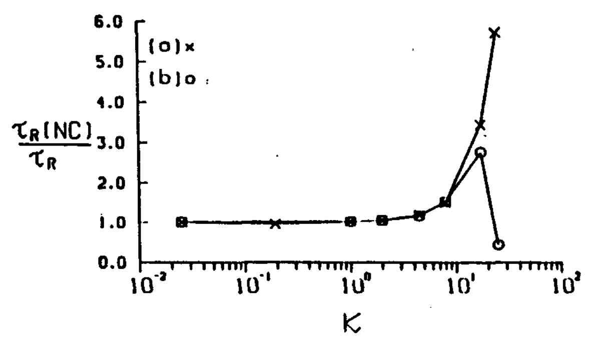

Fig.4 The ratio of τ_R(NC) used in the numerical solution of Liouville equation (1) to τ_R as obtained by the two methods described in the text, as function of κ. (a) A NLLMS fit to expressions of Ref.9 (X) ; (b) On the basis of the linewidths that were obtained by NLLMS to two arbitrary Lorentzians and expressions given by Hubbards (8) (O)

As expected for $\kappa<2$ the values recovered differed from τ_R(NC) by less than 2%. On the other hand for $2 \leq \kappa \leq 25$ the recovered parameters by the NLLMS differed significantly from τ_R(NC). These results give an error up to an order of magnitude in τ_R. The different approach of the two methods indicates that the difference between the results they give is an indicator for slow motion.

For two exchanging sites Westlund and Wennerström (7) obtained under SOPT an analytical expressions for the lineshape. We compare their expressions, with the numerical solutions of the Liouville equation (1) under conditions [2] that include slow motion for one of the two sites. It can be shown on the basis of Eqs.25-26 of Ref.1 that for fast rotational motion ($\kappa_J<1$) an analytical expression similar to SOPT expressions is valid with τ_R^J being replaced with τ_c^J defined in Eq.[3]:

$$\frac{1}{\tau_c^J}=\frac{1}{\tau_R^J}+\frac{1}{\tau_J} \qquad [3]$$

as an effective correlation time. This makes the SOPT expressions valid for $\kappa_J \leq 1$ (fast motion) and over the extended range of exchange times that includes $\tau_J \leq \tau_R^J$. However, the fit between the results of the numerical calculation and the extended SOPT expressions becomes worse as the motion is getting slower ($\kappa_S>1$). A plot of FIT vs. λ (Fig.5) gives a maximum.

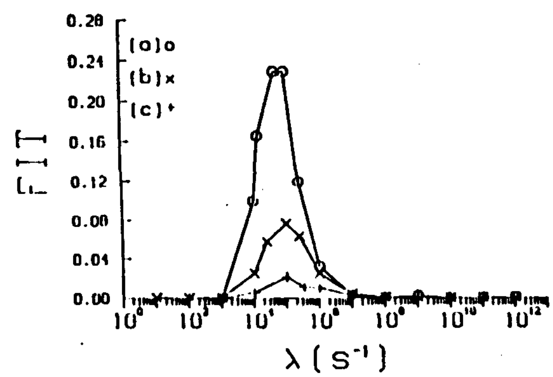

Fig.5 FIT vs. λ χ_S=2MHz (a) $\tau_R^S=2.25\times10^{-5}$s (O) ; (b) $\tau_R^S=4\times10^{-6}$s (X) ; (c) $\tau_R^S=9\times10^{-7}$s (+)

This dependence on λ can be rationalized as follows: At slow exchange both sites are relaxing independently and since the bound state which contain only the minority of the spins, it gives a negligible contribution to the total lineshape. As the exchange rate (λ) increases, the dynamics of the slow rotating site is becoming the dominant process in the relaxation of all the spins, causing FIT to increase. As soon as τ_c^S is predominantly determined by τ_S and one has $\kappa_S\tau_c^S/\tau_R^S<1$ so that reorientation is determined by the fast moving site and the exchange process, causing motional narrowing. In order to summarize the effect of slow rotational motion on FIT we plot in Fig.6 FIT(max) (maximum of FIT taken over the range of exchange times in [2]) as a function of κ_S. The figure shows that FIT(max) increases with κ_S and, for $\kappa_S=1$, reaches a value of about 0.02 which can be detected experimentally. The slower the rotational motion gets the bound site is approaching rigid limit and FIT(max) reaches asymptotic value which depends on the fraction of the slow moving site.

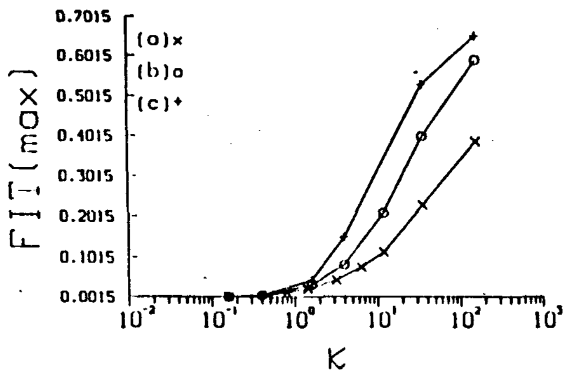

Fig.6 FIT(max) vs. κ_S. (a) χ_S=2MHz, P_S=0.01 (X) ; (b) χ_S=10MHz, P_S=0.01 (O) ; (c) χ_S=10MHz, P_S=0.28 (+)

C. Analytical expression for the longitudinal relaxation with unrestricted amount of spins in the slow motion limit.

Longitudinal relaxation for slow and fast moving spins is calculated by methods similar to the ones described in Ref.1 for the transverse relaxation. We find it to be adequatly described by the limit of SOPT. Since the exchange process is assumed to be independent of spin state it seemed to us that the expressions of ref.7 for the lineshape can be adopted to give the Fourier transform of z-component of the magnetization for exchanging spins. This leads to:

$$M_{Iz}(\omega)= 2M_0 \operatorname{Re} \sum_{k=1}^{2} k^2 \frac{P_I(r_{kS}+i\omega+P_S\lambda)}{(r_{kS}+i\omega)(r_{kI}+i\omega)-P_S P_I} \quad [4]$$

with:

$$r_{kS}=0.4\pi^2 \chi_S^2 J_k+P_I\lambda$$

$$r_{kI}=0.4\pi^2 \chi_I^2 J_k+P_S\lambda$$

$$J_k=\frac{\tau_R}{1+(k\omega_0\tau_R)^2}$$

Similarly one obtains M_{Sz} by interchanging indices I and S. To verify Eq.4 we solved the Liouville equation in Ref.1 for the longitudinal relaxation and compared the results with Eq.4. We got a good fit (FIT<0.006) for any ratio of concentration of the two sites.

Conclusion

For systems containing a site which is in the slow motion limit a good fit to biexponential decay may be obtained, though the widths (decay rates) and second order shifts may differ significantly from their values as predicted by SOPT, and one should analyze the experimental results using a fit to models that are based on numerical solutions of the Liouville Equation.

References

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