*NMR Imaging Methods Seen as Trajectories in the Reciprocal Space**

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During the last decade, several methods for proton NMR imaging of the human body were proposed and have been tested in various laboratories. The most efficient of these are the ones where the signals of a full slice, or even volume, are measured simultaneously. In this paper a rather general description of NMR imaging is presented. For reasons of simplicity, the description is limited to two dimensions only. A 2-dimensional case can be realized by selective excitation of a slice.

In many 2-dimensional methods, the excitation of the spins in the slice is followed by the application of constant or time-dependent homogeneous gradients in the main magnetic field. Such methods are, for instance, Projection Reconstruction, Fourier Zeugmatography in its various forms, and Echo Planar Imaging. They are all covered in the following description.

Let f(x,y) be the function to be imaged, where x and y are the spatial coordinates of the slice. In simple cases this function is the spin density, but often it will also reflect the local relaxation times. In general, f(x,y) represents the nuclear magnetization distribution at a certain instant.

For the description of the methods, we introduce $F(k_x,k_y)$ as the 2-dimensional Fourier transform of f(x,y), i.e.

$$F(k_x,k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\exp[i(k_x x + k_y y)]dxdy.$$

The imaging methods mentioned above correspond to various ways of determining the function $F(k_x,k_y)$. As soon as $F(k_x,k_y)$ is known to a certain extent, f(x,y) will be known within corresponding limits.

In a simple form of NMR imaging, a constant gradient **G** is applied directly after the excitation, and the free induction decay signal is double-phase-sensitively detected, to give the complex low frequency signal S(t). It is not difficult to show that this signal simply *is* the function $F(k_x,k_y)$ on a straight line k_u through the origin in the k_xk_y-plane. (Here, effects of T_2 decay during detection are neglected.) This line k_u makes the same angle with k_x as **G** does with respect to x. The correspondence between k_u and t is given by $k_u = \gamma Gt$, where γ is the gyromagnetic ratio. In other words, the detection of the signal S(t) corresponds to probing $F(k_x,k_y)$ on a trajectory in k space that is determined by the equation of motion $d\mathbf{k}/dt = \gamma\mathbf{G}$.

This statement also holds if time-dependent gradients are applied. So in general we have $d\mathbf{k}/dt = \gamma\mathbf{G}(t)$. A change in the direction of **G** thus results in a corresponding modification of the direction in which the trajectory is followed, while a change in the magnitude of **G** affects the speed along it.

Descriptions of NMR imaging in terms of k-space trajectories have also been given by others (1,2) and might be a promising approach for finding new methods.

It is of special interest to find methods that have a considerably shorter measuring time than Fourier Zeugmatography and yet produce good-quality images. However, due to the dephasing effects of the unwanted inhomogeneities of the magnetic field, most trajectories cannot straightforwardly lead to an acceptable method.

As an example of a new practical method, we shall describe one particular trajectory in detail. Just as in Fourier Zeugmatography, the function $F(k_x,k_y)$ is measured on a rectangular grid, along lines parallel to k_x. In Fourier Zeugmatography, only one such line is measured after each

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excitation pulse. In the new method, several adjacent lines, e.g. 2, 3, or even more, are measured after an excitation pulse. This is achieved by means of a sinusoidal trajectory. In a limiting case, this method becomes equivalent to a version of the echo planar imaging method.

We finally note that the description can readily be extended to three dimensions.

REFERENCES

[1] R. S. Likes, Moving gradient zeugmatography, European Patent Application, Publication number 0,024,640, Date of publication 11.03.81.

[2] D. B. Twieg, Generalized Fourier nuclear magnetic resonance (NMR) imaging methods, *Proc. S.P.I.E. (New Orleans, LA, USA, May 9-12, 1982)*, Vol. 347, pp. 354-64.